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A NOTE ON KELSEY'S "THE ECONOMICS OF CHAOS OR THE CHAOS OF ECONOMICS"

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IN A FINE introductory piece of deterministic chaos and its relationship to economic modelling that recently appeared in this journal, David Kelsey (1988, pp. 36–37) wrote that:

> Most of the [chaos] studies which have been carried out have had physical applications in mind .... Most physical systems are dissipative, i.e, the system contains forces due to friction .... Another possibility is that the principle of conservation of energy can be invoked to require that a ... system be conservative .... Economic systems cannot in general readily be classified as dissipative or conservative since there is no concept which corresponds to energy.

The purpose of this note is to draw a distinction between the thermodynamic conception of a dissipative system and its formal, mathematical, definition and to argue that, in terms of the latter, economic systems can indeed be classified as dissipative.

Following Mosekilde, Aracil and Allen (1980, p. 20), the thermodynamic conception of a dissipative dynamical system is one of a system that, over time, has its high quality energy transformed into low quality energy as a result of its interaction with some sort of resistance (e.g., friction). This is clearly the view of dissipative systems taken by Kelsey. In more general mathematical terms, however, a dissipative system is defined to be *any* dynamical system of equations whose divergence of flow is negative. Specifically

$$\begin{aligned} \dot{x} &= P\{x(t), y(t), z(t), \ldots\} \\ \dot{y} &= Q\{x(t), y(t), z(t), \ldots\} \\ \dot{z} &= R\{x(t), y(t), z(t), \ldots\} \\ &\vdots \end{aligned} \tag{1}$$

and

$$\operatorname{div}\{\dot{x}(t), \dot{y}(t), \dot{z}(t), \ldots\} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} + \ldots < 0 \tag{2}$$

where $x(t), y(t), z(t), \ldots$ are the system's (state (or stock) variables and $\dot{x}(t), \dot{y}(t), \dot{z}(t), \ldots$ are the system's net rate of flow variables.

The interpretation of the divergence criteria of equation (2) is that on net, over time, a dissipative system releases into its environment some portion of the elements that define its dynamic state. This being the case, economic systems must clearly be classified as dissipative for they exhibit this very behavior: capital stocks depreciate, people (labor stocks) migrate and die, firms ship widgets to customers from their stocks of inventory, and producers and consumers form expectations by smoothing information. To be fair, it must be pointed out that the legitimacy of this generalization from physical to social systems, beyond mere mathematical analogy, is still under debate (see the discussions in Mosekilde, Aracil and Allen 1980, p. 20 and Prigogine and Stengers 1984).

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